

Indecomposable semiinfinite string-localized positive energy matter and "darkness"

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March 2008

Abstract

In the absence of interactions indecomposable positive energy quantum matter comes in form of three families of which the massless so called "infinite spin" family which appeared first in Wigner's famous 1939 work is (if mentioned at all) usually dismissed as "unphysical" but without indicating what principle (if at all) is being violated.

Using novel methods which are particularly suited for problems of localization, it was shown that these representations cannot be generated by pointlike localized fields but rather require the introduction of string-like generators which are localized along semiinfinite spacelike strings. We argue that such objects can neither be registered in quasilocal Araki-Haag counters nor pair-produced from standard matter.

Reviewing the mathematical status of Murphy's law in local quantum physics (everything which is not prohibited to couple does indeed couple) off and on shell, we are led to the result that perfect darkness is only possible in QFT with string-localized fields.

Pacs. 95.35+d, 11.10-z, 11.30-Cp

1 Localization and darkness

One of the most enigmatic particle physics enrichments coming from astrophysical observations is the discovery of "dark matter" in the halos of galaxies whose contribution to the total matter/energy content of the universe is a multiple of standard matter. Here by standard matter we mean interacting theories which are generated by point-like localized fields and which have positive energy particles of finite spin (helicity) as classified by Wigner [1]. Even if this new form of matter is not totally inert relative to standard matter, the consistency with the data demand that at least its coupling to be weak. The most popular proposals with the least amount of new parameters came from chargeless components of

supersymmetric extensions of the standard model¹ (neutralinos, WIMPS). Such models lead to measurable production rates of dark matter through high energy collisions of standard matter, and related experiments are well on their way.

In this paper the main aim will be to explore the possibility of existence of perfect dark matter (pDM) in coexistence with standard matter within one model of local quantum physics. The minimal requirement for perfect darkness is that the production rate from standard matter vanishes. In a later part of this article we will also contemplate the possibility that an appropriately defined algebraic *subspace of counters* (a subspace of the operator algebra of QFT) for standard matter is inert to pDM. Since the pDM is gravitating, it is not totally decoupled from standard matter; however we are here primarily interested in a scenario where even without gravity the word would not tensor factorize in a standard and a dark part.

Needless to say that pDM will gain astrophysical interest only if those DM production experiments lead to a null-effect. But since the conceptual setting is tangent to the core of (especially non Lagrangian) QFT, there is also some purely theoretical interest even if the here proposed explanation for darkness will be ruled out by non-vanishing production rate.

It has been known for a long time that properties of confinement and darkness, which in relativistic QM would be phenomenologically accounted for by confining potentials or by decoupling production channels, lead to extremely nontrivial structural problems within local quantum physics.

The main obstacle is the principle of causal locality which has the tendency of coupling all localized states with each other. This state of affairs is sometimes referred to as *Murphy's law of local quantum physics*: if localized states can be coupled (subject to their superselection rules), they will couple.

As the various sociological versions of Murphy's law, also this one has a metaphoric connotation since it only expresses an unexpected, ill-understood tendency. There are various mathematical theorems which may be considered as its rigorous meaning. One is the Reeh-Schlieder theorem [2] which implies that by operations in a bounded spacetime regions any state in the universe can be approximated. In fact not only the vacuum, but any finite energy states is coupled to the rest of the world by any localized algebra, as small as its localization region may be.

A closely related structural property is the impossibility to decouple observables localized in a region \mathcal{O} from those localized in the causal disjoint region \mathcal{O}' . A special case which shows that in a local positive energy QFT one cannot decouple a localized projector $P(\mathcal{O})$ from its spacelike separated translate is Malament's theorem [4]. Although not directly related to the issue of this paper (but related to Malament's theorem) we mention in passing that the perhaps most spectacular disparities between relativistic² QM and local quantum

¹For reasons which we have explained in [3] we do not believe that "unparticles" can be a serious contender for DM.

²By relativistic QM we mean the Poincare representation theoretical setting of *direct particle interactions* which leads to a Poincare invariant clustering S-matrix [5] but cannot implement the local covariance principle which is characteristic for QFT.

physics (LQP) shows up in the two different notions of entanglement based on the two different localization concepts (the Born-Newton-Wigner localization in QM and the modular localization in LQP [6]). The insufficient distinction between the information theoretic quantum mechanical entanglement from the thermal manifestation [7] (localization entropy [8]) has been the cause of many confusions, including those behind the information loss in black hole physics.

In some way the split property [2], by which one can enforce a tensor factorization after *separating \mathcal{O} from \mathcal{O}' by creating a attenuation distance e for the vacuum fluctuations $\mathcal{O}'_e \subset \mathcal{O}'$* [8], restores some of the properties of QM. but the result is still not the ideal world of QM since all states which are relevant in particle physics (vacuum, finite energy states) become *thermally entangled* states with respect to a new Hamiltonian associated with the localization region. On the other hand the factorizing states have unbounded energy and particle number and hence lack a clear physical interpretation in terms of particles.

The root of all these unusual manifestations is the radically different structure of the local algebras: whereas in (relativistic) QM the B-N-W localized algebras are algebras of bounded operators $B(H)$ in a factor Hilbert space, the modular localized algebras are copies of the monad algebra (the unique hyperfinite factor of type III_1) [7]. This algebraic distinction accounts for all the structural differences.

Hence in contrast to Murphy's law in everyday life, its LQP atavar is not only the cause of complicating the quantum mechanical life by coupling infinitely many channels, but it also creates that conceptual tightness which accounts for the fact that we consider QFT in comparison with relativistic QM as the more fundamental setting. The before mentioned Leibnitz picture of reality in LQP which states that the complex structure of QFT can be encoded into the relative positioning of a finite number (2 in chiral models, 3 in $d=1+2$, 6 in $d=1+3, \dots$) of structureless "monads" without any individuality (isomorphic to the unique hyperfinite III_1 factor) is inexorably linked to the apparent messy aspects which LQP presents from a quantum mechanical setting.

Having exposed some of the conceptual problems which one has to confront before envisaging darkness in the above perfect sense, we now ask the more concrete and observational relevant question *whether the S -matrix can escape the on shell adaptation of Murphy's law*. Using on-shell analytic properties which extend those which one can derive in the presence of interpolating local fields (in particular the crossing relation), one can show that a pure elastic scattering without the participation of creation channels is impossible. This nonperturbative assertion known as Aks' theorem [9], which with the passing of time has acquired the Rumsfeldian status of an "unknown known", is limited to 4 and higher dimensions. In lower dimension, in particular in $d=1+1$ there is a subterfuge which is related to the Coleman-Mandula issue arising from infinitely many conservation laws. The localization aspect behind this mechanism is the existence of rather well-behaved ("temperate") vacuum-polarization-free one-particle generators (PFG's) of the wedge algebra in $d=1+1^3$ [10].

³Any smaller localization region does not permit the existence PFG's in the presence of

The Aks' result would in particular exclude a vanishing production rate of DM from standard matter, at least in case that the DM is (as the SM) described by pointlike localized fields. But the principles of QFT permit field algebras which cannot be generated by fields which are better localized than semiinfinite string-like. Here the observables are always assumed to be pointlike generated i.e. the candidates for string-like localized generating fields⁴ carry nontrivial "Casimir charges" which distinguish the string-generated sector from the vacuum sector.

String-like localization is the most general localization which follows from a mass gap spectrum and the derivation of this fact by Buchholz and Fredenhagen [12] belongs to one of the most profound conceptional enrichments of QFT. The stringlike localization destroys the possibility of describing the same out/in configuration by the $t \rightarrow \pm$ limits of the same wave-packet smeared Heisenberg field; rather the string direction in both cases must be chosen different in order to obtain the same out/in configuration [12]. With the breakdown of crossing the main obstacle leading to the Aks theorem has been taken out and with it the on-shell reign of Murphy's law has been weakened. In fact in the conclusions of [12] one encounters the remark that a hypothetical string quark matter, once bound into hadrons or lost into the surrounding space, cannot be produced because the string-localization may impede the production process. Nowadays we would think that this idea works better with DM.

Semiinfinite string-like localization may be incompatible with Lagrangian quantization, but it is the kind of localization which can be rigorously derived from the mass gap assumption in conjunction with local observables. In such a setting the full field algebra is generated by string-localized (basic) fields which contains a subalgebra which is generated by pointlike composites. In order to create all the particles from the vacuum one needs the string fields which in some way play a similar role as the interpolating fields in the LSZ scattering theory⁵. localization which follows from massive theories with a mass gap. There exists at present no model which requires string-localized fields, but given the lack of nonperturbative knowledge even with respect to point-like localized Lagrangian models, their existence can hardly be doubted. The observable imprints such stringlike fields leave on the level of particles and their interactions are rather subtle; the particle spectrum with gaps just looks the same as for point-like fields and the observable content of a breakdown of the analytic aspects of on-shell crossing⁶ is hard to assess. However the main ingredient into Aks' proof

(any) interaction.

⁴It is regrettable that the good autonomous use of this terminology always requires one to add that its use in string theory is totally metaphoric; e. g. the N-G quantum object is a generalized free field [11].

⁵Strictly speaking the stringlike localization destroys the possibility of describing the same out/in configuration by the $t \rightarrow \pm$ limits of the same wave-packet smeared Heisenberg field; rather the string direction in both cases must be chosen different in order to obtain the same out/in configuration [12]. This is at the basis of the previously mentioned breakdown of crossing.

⁶The conceptual change is however quite drastic: the uniqueness argument in [13] for the inverse scattering which uses crossing will break down. The one S-matrix one QFT inverse

is the validity of crossing and the absence of certain creation channels would perhaps constitute the most dramatic manifestation of the presence of string-like generators.

An interesting development which presently drives a good part of structural research about localization properties and in particular the use of lesser localized objects as wedge-localized generators is the interesting fact that among those objects one can find generators with significantly simpler properties [14] than among interacting point-like fields. The latter create, as already alluded to before, in the presence of (any) interactions upon application onto the vacuum very complicated infinite vacuum polarization clouds interaction induced. In this way the first hard mathematical existence theorems about an interesting strictly renormalizable (beyond the superrenormalizable models of the 60ies) models have been proven and the (generally hard) problem of asymptotic completeness was established [15]. The mathematical concepts applied to $d=1+1$ has not yet led to an existence proof of higher dimensional QFT but an interesting link with attempts to construct nonlocal QFT of the Moyal deformation kind has arisen [16]. This is another attempt to cut a breach into the territory governed by the on-shell Murphy's law.

In this note we want to direct the reader's attention to an even more radical type of perfectly invisible matter i.e. quantum matter which remains either totally or partially inert relative to standard matter. As all positive energy matter it has a gravitational manifestation. But different from standard matter, and also from the previously presented massive strings, its semiinfinite string nature of its field generators leads to indecomposable semiinfinite string-localized states. Clearly this is only possible in the presence of zero mass; indeed such irreducible zero mass representation do occur in the Wigner's classification of indecomposable positive energy solutions and in the next section we will remind the reader of their properties. This extreme form of hypothetical matter cannot even trigger Araki-Haag counters [2]. A-H counters are counters of localizable quantum matter represented by operators which do not respond to the vacuum but register to excitations above the vacuum; as a result of the Reeh-Schlieder theorem they cannot be strictly local but they are contained in the subalgebra of *quasilocal* operators i.e. operators which admit a *rapid* approximation by local operators⁷.

2 String-localized states from representation theory

The role of semiinfinite string localization as the best generators for those representations of the Poincare group which are inconsistent with point-like gener-

scattering statement has been first observed in factorizing models. The validity of crossing extends the uniqueness (but not the existence) to higher dimensions.

⁷The analysis with A-H counters which leads to a generalization of the Wigner particle theory ("particle weights" [17]) is primarily a theoretical concept since it ignores the fact that the final registration needs the intervention of light emitted by charged particles.

ators was not seen for a very long time. This had two quite interesting historical reasons. On the one hand Wigner used a localization concept (the Born-Newton-Wigner localization) which is obtained from quantum mechanics by adjusting the localization operator \vec{x}_{op} to the L-invariant inner product, thus maintaining the probability aspect and its relation to projectors in the relativistic context but lacking local covariance; in a quantum theory without a maximal velocity the independence from a reference frame is only restored asymptotically between asymptotic timelike separated events⁸ whereas the principle of local covariance in QT with a maximal velocity requires the *modular localization* (which Wigner missed and which will be explained below). So on the bright side the two important but very different localization concepts coalesce asymptotically, a fact which is crucial for the Poincare invariance and the probability interpretation of the S-matrix. Although particles in interacting QFT do not exist, their asymptotic appearance accounts for the observable richness of particle physics.

Besides Wigner's missing the appropriate covariant localization concept for finite propagations, there is a second reason why string localization posed a difficult conceptual hurdle. QFT was discovered by Jordan in the form of quantization i.e. a parallelism of the classical field formalism. But it turns out that this parallelism is limited to pointlike fields *since indecomposable string-like quantum states have no classical analog and classical relativistic strings have an associated QFT which is not string-localized in any intrinsic quantum sense*. This can be explicitly illustrated by realizing that on the one hand the class of infinite spin representations do not arise from a classical Lagrangian and on the other hand To illustrate this point in a completely explicit manner: the Wigner infinite spin representation does not arise from quantizing a classical Lagrangian and the quantum object associated with the classical Nambu-Goto string is not a quantum string-localized field but a point-like so-called generalized free field [11] i.e. the interpretation of this localization point as the center of mass point of a quantum string is a metaphoric invention which ignores the intrinsicness of the quantum localization concept⁹.

Wigner found that there are precisely three families of indecomposable positive energy representations of the Poincare group, two rather large families containing continuously many inequivalent representation and one representation family which is of countable cardinality. They are distinguished by the nature of the little group and its representation theory.

Besides the best studied massive family, for which the little group is the invariance group of a timelike vector (and hence isomorphic to $SO(3)$), there exist two massless families whose little group of a lightlike vector is isomorphic to a noncompact euclidean subgroup group of the Lorentz group $E(2) \subset L(3, 1)$.

⁸This fact is of crucial importance for the S-matrix in relativistic QM and for the probability interpretation of the S-matrix in terms of cross sections.

⁹Not all treatments of the quantum N-G model have followed that metaphor suggested by Euclidean functional representation. In [18] the quantization was done by using the complete integrability of the model and it was later shown that the resulting theory is not equivalent to the canonically quantized one [19].

Since the representation of the P-group is induced from $E(2)$, this property is passed on to the P -representation.

What distinguishes the two massless families is the *nature of the $E(2)$ representations*; whereas the finite helicity family which contains the known zero mass particles is a degenerate representation in which the euclidean translation is represented trivially (which compactifies the representation despite the non-compactness of the group), the third family results from a faithful $E(2)$ representation which preserves the group theoretic noncompactness and comes with unusual, conceptually challenging properties. The little Hilbert space is now an infinite dimensional space of Fourier components which describe an $E(2)$ -irreducible infinite intrinsic abelian angular momentum tower; this is why we prefer the terminology "infinite spin" over Wigner's "continuous spin" (which refers to the continuous values of the Casimir invariant). The appearance of this infinite tower prevents the extension of the P-group to the conformal group despite the vanishing of the mass.

These positive energy representation of the third kind had a long and complicated history; most particle theorists, including Wigner and later Weinberg felt that there is something unphysical about these representation i.e. a conceptual reason why nature apparently only uses the two other classes of positive energy representations but not this one. The first theorem showing that these representations do not allow pointlike field generators appeared in 1970. The problem of whether these representations have some residual localization lay dormant up to recently [20] when it was shown that spacelike cone localization of states is a consequence of the positive energy condition¹⁰. It turned out [21] that the local covariant generators of such representations are string-localized fields $\Phi(x, e)$ localized on semiinfinite spacelike lines $x + \mathbb{R}_+ e$, $e^2 = -1$

The mathematical framework of the relevant quantum localization concept is fairly new and goes under the name of *modular localization* [14][20].

Since in the deafening noise of present particle physics fashions progress on old conceptual problems are hardly noticed, we sketch the main idea of modular localization without proof in the simplest spinless case (where also traditional methods would be sufficient) and only quote the results for the case at hand. For more details we refer to the mentioned literature.

Intuitively modular localization is the quantum counterpart of causal localization in classical field theories with a maximal propagation speed. It is inherent in relativistic QFT and its structural quantum properties become more exposed after one liberates it from the use of particular field coordinatizations. In other words modular localization is the standard localization implemented by (necessarily singular) field coordinates in relativistic QFT after one separates the unique localization concept from the highly non-unique coordinatization-dependent aspects of field generators. Hence modular localization is a property

¹⁰The reader will notice a similarity to the B-F theory of massive strings. But the latter are operator strings whose action on the vacuum creates massive states which have a pointlike decomposition theory under the Poincare group whereas the massless third kind Wigner states are indecomposable string-like i.e. the state decomposition theory goes beyond the generating theory of operator algebras.

of the system of local algebras which does not depend on which field among the infinite number of possible field coordinatizations one selects.

Here we are interested to localize states in a Wigner representaion. Starting from a space of wave functions of a scalar particle one first defines two commuting operators in H_{Wig} which are associated to the $t - x$ wedge $W_0 = \{x \mid x_1 > |x_0|\}$, namely the unitary representers \mathbf{u} of the wedge-preserving Lorentz boost $\Lambda_{W_0}(\chi)$ which commutes with the antiunitary representer $\mathbf{u}(j_{W_0})$ of the wedge-reversing reflection j_{W_0} across the edge of the wedge (third line).

$$\begin{aligned} H_{Wig} &= \left\{ \psi(p) \mid \int |\psi(p)|^2 d\mu(p) < \infty \right\} \\ (\mathbf{u}(\Lambda, a)\psi)(p) &= e^{ipa} \psi(\Lambda^{-1}p), \\ (\mathbf{u}(j_{W_0})\psi)(p) &= \overline{\psi(-j_{W_0}p)} \end{aligned} \quad (1)$$

One then forms the¹¹ “analytic continuation” in the rapidity $\mathbf{u}(\chi \rightarrow -i\pi)$ which leads to unbounded positive operators. Using the customary notation in modular theory, one passes to the following unbounded closed antilinear involutive operators in H_{Wig1}

$$\begin{aligned} \mathfrak{s}(W_0) &:= j_{W_0} \delta_{W_0}^{\frac{1}{2}}, \quad \delta_{W_0}^{it} := \mathbf{u}_{W_0}(\chi = -2\pi t) \\ (\mathfrak{s}(W_0)\psi)(p) &= \psi(-p)^*, \quad \text{dom } \mathfrak{s}(W_0) = \text{dom } \delta_{W_0}^{\frac{1}{2}} \end{aligned} \quad (2)$$

where the analytic properties of the domain of this unbounded modular involution $\mathfrak{s}(W_0)$ with $\mathfrak{s}^2(W_0) \subset \mathbf{1}$ consists precisely of that subspace of Wigner wave functions which permit that analytic continuation on the complex mass shell which is necessary in order to get from the forward to the backward hyperboloid ($\chi \rightarrow \chi - \pi i$). The main assertion of modular localization is that the ± 1 eigenspaces (real since $\mathfrak{s}(W_0)$ is antiunitary) are the real closed component of the dense $\text{dom } \mathfrak{s}(W_0)$

$$\begin{aligned} \mathfrak{K}(W_0) &= \{\psi \mid \mathfrak{s}(W_0)\psi = \psi\}, \quad \mathfrak{s}(W_0)i\psi = -i\psi \\ \text{dom } \mathfrak{s}(W_0) &= \mathfrak{K}(W_0) + i\mathfrak{K}(W_0) \\ \mathfrak{s}(W_0)(\psi + i\varphi) &= \psi - i\varphi \end{aligned} \quad (3)$$

The dense subspace $\text{dom } \mathfrak{s}(W_0)$ (i.e. $\overline{\text{dom } \mathfrak{s}(W_0)} = H_{Wig}$) is precisely the one-particle component of the W_0 localization space associated with a scalar free field $A(x)$, or in terms of the real subspace¹²

$$\mathfrak{K}(W_0) = \text{clos} \{(A(f) + A(f)^*)\Omega \mid \text{supp } f \subset W_0\} \quad (4)$$

but the modular construction of localized subspaces avoids the use of singular field coordinatizations and the ensuing smearing with classically localized test

¹¹The unboundedness of the \mathfrak{s} involution is of crucial importance for the encoding of geometry into domain properties of unbounded operators.

¹²The closedness of K does not imply that of $K + iK$.

functions and relies instead on the more intrinsic description in terms of domains of distinguished unbounded operators in the ¹³ Wigner space associated with the representation $(m, s = 0)$. The second line is the defining relation of what is called a *standard real subspace* of a Hilbert space. The *standardness property* is equivalent to the existence of an abstract (nongeometric) modular involution.

Applying Poincaré transformations one generates from $\mathfrak{s}(W_0)$ and $\mathfrak{K}(W_0)$ to the W -indexed families $\{\mathfrak{s}(W)\}_{W \in \mathcal{W}}$, $\{\mathfrak{K}(W)\}_{W \in \mathcal{W}}$. The localization spaces for smaller causally complete spacetime regions \mathcal{O} (which could be trivial) are obtained by intersections $\mathfrak{K}(\mathcal{O}) = \cap_{W \supset \mathcal{O}} \mathfrak{K}(W)$. A remarkable property of all these spaces resulting from Wigner's positive energy representation setting is the validity of *Haag duality*

$$\mathfrak{K}(\mathcal{O}') = \mathfrak{K}(\mathcal{O})' \quad (5)$$

where the dash on the region denotes the causal complement and that on the K -space stands for its symplectic complement within H_{Wig} i. e. $Im(K, \varphi) = 0$ for all $\varphi \in \mathcal{K}(\mathcal{O})' = j_{\mathcal{O}} \mathfrak{K}(\mathcal{O})$

The final step is the functorial ascend to the net of spacetime localized operator algebras in the Wigner-Fock space $H_{W-F} = H_F(H_{Wig})$ (with creation/annihilation operators $a^*(p), a(p)$)

$$\begin{aligned} Weyl(\psi) &= \exp i(a(\psi) + a^*(\psi)), \quad \psi \in \mathfrak{K}(\mathcal{O}) \\ \mathcal{A}(\mathcal{O}) &:= alg \{Weyl(\psi) \mid \psi \in \mathfrak{K}(\mathcal{O})\}, \quad \mathcal{A} := \cup_{\mathcal{O}} \mathcal{A}(\mathcal{O}) \\ K(\mathcal{O}) &= \overline{\{(A + A^*)\Omega \mid A \in \mathcal{A}(\mathcal{O})\}}, \quad \mathfrak{K}(\mathcal{O}) = P_1 K(\mathcal{O}) \end{aligned} \quad (6)$$

where *alg* denotes the operator (von Neumann) algebra generated by the unitary Weyl operators in the Wigner-Weyl space and P_1 is the projection of the Wigner-Fock space onto the Wigner one particle space. Note that there are no spacetime dependent field coordinates, the construction is as intrinsic and unique as the Wigner representation theory.

This modular construction exists for all three Wigner representation families. The $\mathfrak{K}(\mathcal{O}) + i\mathfrak{K}(\mathcal{O})$ spaces for $\mathcal{O} = \mathcal{D}$ = double cone (the prototype of a simply connected causally complete compact region) for the first 2 families are dense in H_{Wig} whereas the third kind of Wigner matter yields a vanishing $\mathfrak{K}(\mathcal{D})$ for double cones. In that case the nontrivial space with the tightest localization $\mathfrak{K}(\mathcal{C})$ is associated with an (arbitrarily thin) noncompact spacelike cone $\mathcal{C} = x + \mathbb{R}_+ \mathcal{D}$ with apex x and an opening angle which is determined by \mathcal{D} . All relations about \mathfrak{K} pass to the K 's in Wigner-Fock space.

There is no problem in adapting the modular setting to the presence of interactions; however different from the free situation there are no one-particle creators in compactly localized algebras (for details see [10]).

The steps explained above in the spinless context can be carried out for the first two families with the help of intertwiners. These can also be constructed

¹³It was precisely this uniqueness which was Wigner's main motivation for bypassing the confusing plurality of the quantization setting (many different equations of motion with the same physical content) in favor of an intrinsic description. However the adaptation of the Born particle localization (the Newton-Wigner localization) was taking him away from covariant causal locality.

without modular theory by standard group theoretical techniques as explained in Weinberg's first volume of [22]. They intertwine the unique Wigner representation to the denumerable infinite set of $(2A + 1)(2\dot{B} + 1)$ component *spinorial fields* indexed by $r = (A, \dot{B})$

$$\begin{aligned} \Phi_r(x) = & \sum_{k=-s}^s \int d\mu(p) \{ e^{ipx} u_{k,r}(p) a^*(p, r) + \\ & + e^{-ipx} u_c(p)_{k,r} b(p, r) \}, \quad |A - \dot{B}| \leq s \leq A + \dot{B} : \end{aligned} \quad (7)$$

But only in the massive case the full spinorial range (A, \dot{B}) relative to the given Wigner spin s is realized; the massless case, as a result of its different little group requires in addition $A - \dot{B} = 0$. This generates big gaps in the full spinorial spectrum¹⁴. In particular there is no covariant vectorpotential for $s = 1$ and no metric potential $g_{\mu\nu}$ for $s = 2$; in both cases one has to go to higher degree tensor fields than in the massive case (the field strengths $F_{\mu\nu}$ and the linearized Riemann tensor $R_{\mu\nu\kappa\lambda}$) in order to remain within quantum physics. On the other hand a covariant semiinfinite *string-localized* vector potential $A_\mu(x, e)$ or metric potential $g_{\mu\nu}(x, e)$ poses no problems i.e. the missing possibilities in the spinorial formalism can be filled with string-localized field generators. These covariant string-localized "potentials" associated to pointlike "field strengths" possess mild short distance property (scale dimension is one).

For Wigner's *third kind of matter* the only known systematic construction is one which determines a continuous (α -dependent) family of intertwiners $u^\alpha(p, e)$ using their modular localization properties [21][6]. In this way one obtains a continuous set which depend in addition to the momentum p on a spacelike unit vector e , $e^2 = -1$. It intertwines the Wigner transformation, which involves the representation D_κ of the noncompact $E(2)$ little group with the covariance transformation law in p and e and leads to a string field whose intrinsic stringlike extension can be seen by the appearance of a nontrivial commutator if one string enters the causal influence region of the other

$$\begin{aligned} D_\kappa(R(\Lambda, p)) u^\alpha(\Lambda^{-1}p, e) &= u^\alpha(p, \Lambda e) \\ \Psi(x, e) &= \left(\frac{1}{2\pi} \right)^{\frac{3}{2}} \int_{\partial V_+} d\mu(p) (e^{ipx} u^\alpha(p, e) \circ a^*(p) + \\ &\quad e^{-ipx} \overline{u^\alpha(p, e)} \circ a(p)) \\ [\Psi(x, e), \Psi(x', e')] &= 0, \quad x + \mathbb{R}_+ e \not>< x' + \mathbb{R} e' \end{aligned} \quad (8)$$

That certain objects do not admit a pointlike presentation is not limited to these third kind of Wigner (infinite spin) representations. The $d=1+2$ "plektons" (particle associated to braid group statistics) are particles whose field

¹⁴There are more subtle differences to the massive case whose exploration requires more future research: whereas massive representations are Haag dual (5) for spacetime regions of arbitrary higher connectivity, this is not so for the massless representations starting from helicity one HD does not extend beyond simply connected regions.

theoretic description requires spacelike strings [25]. By forming charge-neutral bilinear composites one descends to compactly localizable observables. Last not least the necessity from renormalizability of calculating with string-like potentials instead of point-like field strengths even when the content of a theory can be described in terms of the latter requires a new perturbative technology [23]. The iterative Epstein-Glaser step from the set of n -point time ordered correlators to $n+1$ for rectilinear semiinfinite strings is certainly more involved and without paying attention to the string-localization (only paying attention to the end point) the construction of time ordered correlation functions would lead to insoluble infrared problems.

Interactions of ordinary matter are taking place in a compact spacetime region, at worst they are quasilocal. This places a question mark on whether such matter can interact at all with standard matter and whether it can trigger A-H counters. Leaving the final answer to future more detailed studies there remains the question of whether a pair of such string-particles could be created from a collision of ordinary matter. This should be possible if, as in the case of 3-dimensional anyons [25], the strings are somewhat fictitious analogous to cuts in Riemann surfaces. Otherwise i.e. if the asymptotic directions remain visible, causality should prevent the creation of a pair of strings by colliding ordinary matter. In this case the mechanism for darkness is kinematical and already occurs on a local level unlike the previously mentioned hypothetical B-F subterfuge of the Aks theorem via the breakdown of crossing. For the free field model at hand there are no point-local fields which are quadratic in the creation/annihilation operators as the following calculations shows.

It is important to know whether string-localized operators which applied to the vacuum state create "one-string" states generate algebras which possess point-like generated subalgebras.

The most general field which is quadratic in the annihilation/creation operators and transforms as a scalar is of the form

$$\begin{aligned}
B(x) &= \int \int_{\partial V} d\nu(k) d\nu(l) d\mu(p) d\mu(q) \\
&\quad e^{i(p+q)x} u_2(p, q)(k, l) a^*(p, k) a^*(q, l) \\
u_2(p, q)(k, l) &= \int d^2 z d^2 w e^{i(kz + lw)} F(B_p \zeta(z) \cdot B_q \zeta(w))
\end{aligned} \tag{9}$$

The coefficient function must have the form in the third line; here $\zeta(z) = (\frac{1}{2}(z^2 + 1), z_1, z_2, \frac{1}{2}(z^2 - 1))$ and (z_1, z_2) is the Fourier transformed variables of the variable k describing the space of the little group $E(2)$. The intertwining function u_2 clearly absorbs the complicated Wigner transformation of the creation/annihilation operators and converts it into the simple transformation of a scalar field.

In order to decide the nature of the localization property of $B(x)$ it is not enough to consider its two-point function. According to the Kallen-Lehmann representation its two-point function is automatically causal, but this only means that the distribution-valued state vector $B(x)\Omega$ is point-localized and

implies nothing about the localization of the operator. The string generated algebra would have compactly localized subalgebras in case of existence of tensor fields which are relatively local to the string. In case of our scalar bilinear field B (9) the answer to the question:

$$\exists B \text{ s.t. } \langle q, l | [B(x), \Psi(y, e)] | 0 \rangle = 0, \quad x > y + \mathbb{R}_+ e ? \quad (10)$$

is negative and this is best understood by comparing the contraction functions with those for standard matter. By splitting off a plane wave exponential the matrixelement in (10) only depends on the x - y difference. The Fourier transform of this function is then polynomial in the Fourier momentum and this leads to the spacelike vanishing. The presence of the z, w little-group Fourier transforms in (9) as well as in the definition of $\Psi(x, e)$ indicates a more complicated non-polynomial dependence which after Fourier transform to the relative distance variable $x - y$ has no support properties at all. A more pedestrian way to see this is to restrict the string and B to equal times. This situation cannot be improved by going from bilinear scalars to tensors, or by generalizing from bilinear to $2n$ -linear expressions in the $a^\#$ since by taking different matrixelements the contributions from higher composites can be separated out and the same argument can be repeated. With a probability bordering on certainty the algebras associated to Wigner representations of the third kind do not contain pointlike localized subalgebras and hence no local observables.

But such an illustration of complete absence of local observables in a free field theory can hardly be taken serious as a realistic model for darkness¹⁵. The question is whether such objects can occur *together with standard matter* as part of the same theory id possible by more than the shared coupling to gravity. A structural theorem by Buchholz and Fredenhagen shows that theories with a mass gap may need semiinfinite stringlike generators for the algebras but the application of these noncompact stringlike generators onto the vacuum always creates states which can always be decomposed into pointlike localized states; with other words even though the operator strings cannot be decomposed into compactly localized parts inside the algebra, the states which they create can be decomposed into irreducible massive Poincaré representations. So such strings lead to superpositions of massive states of the standard type which are not dark.

3 Non-gravitational darkness

In the introduction we mentioned a conjecture that massive matter which is string-localized in the B-F sense is outside the range of the on-shell version of Murphy's law (Aks theorem) may show very different behavior from standard matter including darkness in the sense of vanishing production rates. The existence of string-localized zero mass component of states, of which we presented an interaction-free illustration, leads to a more startling kind of darkness. The

¹⁵The total lack of any relation to standard matter would only leave the extremely speculative possibility that the origin of this matter is inexorably linked to that of the still elusive quantum gravity.

idea of relating indecomposable string-localized states with "darkness" is based on the fact that the interaction with normal matter is at least quasilocal i.e. takes place in compact regions. The quasilocality is a compromise one has to make in order to describe counters which do not already click in the vacuum. If one enlarges the local algebras to quasilocal by also allowing operators which can be rapidly approximated by local ones, one finds a dense subspace of quasilocal observables which have no vacuum polarization and which are the objects in terms of which the A-H particle counters are constructed.

The lack of local subalgebras, in particular of quadratic pointlike local composites, dampens hopes that such pure string-localized algebras could account for DM. For whatever DM really consists of, it is certainly not inert with respect to gravity and therefore it should contain a pointlike energy-stress tensor. But the argument is not as cutting as it sounds if one thinks in terms of quantum gravity. If one wants the metric tensor to be an object of direct *quantum* physical significance there is no way around its string-localization since the *quantum* potential of a (linearized) point-like field strength $R_{\mu\nu\kappa\lambda}(x)$ is a string-localized covariant symmetric tensor $g_{\mu\nu}(x, e)$.

As in the analog $s=1$ situation of $F_{\mu\nu}(x)$ and $A_\mu(x, e)$ the problem does not arise in the classical context because there is no quantum imposition from Wigner's representation theory. Whereas in the $s=1$ case one can argue that the vectorpotential is an auxiliary quantity which does not have to be subjected to the quantum positivity requirements as long as one can assure the positivity of a sufficiently large subsystem (which leads the well-known gauge theory setting) this seems to be less palatable for the $g_{\mu\nu}$ in quantum gravity. But then any classical modification of the Einstein-Hilbert equation which involves other combinations of the metric tensor would raise the issue of string-like localization if one starts from a Minkowski background. So even before coupling DM the issue of localization of gravity is not so clear.

Heeding advice from the gauge theory setting one expects the requirement that interacting strings lead to point-like subalgebras to be very restrictive; in fact any result different from an isomorphism between the point-like generated subalgebra in the string-like formalism and the gauge invariant algebra in the gauge setting would be deeply disturbing. Whereas the gauge setting trespasses the positivity of QT but formally keeps the point-like covariant aspect of vectorpotentials and finally arrives at local gauge-invariant fields by a (BRST) cohomological descend, the approach based on string-localized vectorpotentials leads to the same observable point-like generated subalgebra but produces in addition very nonlocal operators which are the interacting versions of massive matter fields which interact with the vectorpotential.

Taking again a lesson from QED one would like to think of the non-local objects as corresponding in the gauge setting to nonlocal gauge invariant operators as the delocalized electric charge-carrying operators. The construction of these operators in the gauge setting is notoriously difficult even in abelian gauge theories since it is not part of the formalism, but rather requires a construction "by

hand" [26]¹⁶. It is well known that electrically charged particles are not Wigner particles but rather *infraparticles* i.e. stable entities which are surrounded by a soft photon halo and which cannot be separated into a hard Wigner part and a soft photon cloud¹⁷. The sharpest possible localization of such infraparticles is semiinfinite stringlike and the "minimal" generator (in the sense of lowest short distance dimension) is formally given by the Jordan-Mandelstam formula of a Dirac spinor localized at the endpoint of the string multiplied by an exponential function of a line integral in terms of the vectorpotential; the difficult perturbative renormalization status has been clarified by Steinmann [26]. The impossibility to read these spacetime formulas as a composite of spinor matter and photon stuff complies with the impossibility to interpret an infraparticle in terms of a mass-shell delta function and a continuum. This form of the spectrum results from the quantum Gauss law which enforces a strong coupling of infrared photons to the charge-current density (the Buchholz theorem [27]).

With string-localized vectorpotentials these delocalization aspects are built into the formalism but there is a prize to pay at another place: one has to understand a new renormalization theory in which the causal relations have to be generalized to strings. Even though it would be enough to know the perturbation theory for vectormesons with a string-fluctuation controlled by localized fixed test function on $d=1+2$ de Sitter space (the range of space-like unit vectors) $A_\mu(x, g) = \int A_\mu(x, e) g(e) de$, the renormalization theory for string-localized fields requires to consider generic directions. These problems are presently under investigation [23].

The QED illustration also focusses attention to the fact that it string-localization in itself does not account for darkness. *Our Leitmotiv is rather that perfect darkness cannot be achieved in point-like QFTs and that there are good reasons for expecting that there are string-localized models with point-like subgenerators in which collision between point-like generated matter does not lead to string-like generated matter.*

This limits the ($s \leq 1$) search for pDM to string-like localized mutually interacting vectorpotentials with nontrivial pointlike generated subalgebras i.e. to the string-localized reformulation of nonabelian gauge theories. It is clear that if one succeeds to formulate renormalization theory for strings (a very nontrivial conceptually and technically challenging step) one will have gained a natural method for the construction of the nonlocal physical operators which correspond to the above Jordan-Mandelstam string objects. The main problem of the standard gauge formulation is that there is no natural way to do this since the gauge formalism is focussed on local observables. In the case of nonabelian interacting

¹⁶For nonabelian gauge theories the inability for doing such "by hand" constructions has been an impediment to explore the physical content beyond the local gauge invariant observables (gluonium,...) which left gluons and quarks in a very opaque status (not very different from pDM).

¹⁷The energy-momentum spectrum is precisely that of "unparticles" i.e. a accumulation of weight starting with a power singularity at the mass of the charged particle which is submerged into the continuum. But charged particles are the most visible "candles" in particle physics, and the conjectured darkness of unparticles is a conceptual illusion caused by insufficient knowledge of QFT and its history.

vectorpotentials one does not expect that the local subalgebra generates the full physical space, rather there should be lots of delocalized stuff in the physical Hilbert space. More concretely, the nonlinear relation of the string-localized vector potentials to point-like observables suggests that the former generate a space which genuinely contains the one generated from the vacuum by the latter. This then could possibly lead to an interacting model in which string-localized matter carrying positive energy could be somewhat out of sight in the sense of this paper. We will come back to this point in a somewhat broader context [24].

Beside the intellectual pleasure derived from the problem whose solution requires a profound knowledge of QFT, the main advantage of the linking dark matter to structural properties of indecomposable string-localized fields is that it allows to make a very precise experimental prediction: there should be no possibility of pair creation from standard matter. In particular it would contradict the predictions coming from supersymmetric extensions of the Standard Model.

A null effect on the other hand would leave no alternative to strings as a result of the Aks theorem for point-like fields based on the validity of the crossing property. In that case the discoverer of the DM Fritz Zwicky and Eugene Wigner, the protagonist of symmetry and of the first intrinsic classification of particles (including irreducible representations which led to the notion of string-like localization) may be linked by more than having been scientists who wrote their important contributions at the same time.

As a theoretical physicist interested in conceptual problems, I always admired Wigner's strict insistence in exploring known principles *before* doing mind games. Whereas the traditional way of valuating observations essentially did not change since the time of Zwicky, the same cannot be said about modern particle theory where the number of researchers following the intrinsic logic of theoretical principles a la Wigner has decreased in favor of those doing mind games.

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